南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

深圳量子科学与工程研究院
Shenzhen Institute for
Quantum Science and Engineering

Weak (anti-)localization
in
topological materials

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南方科技大学

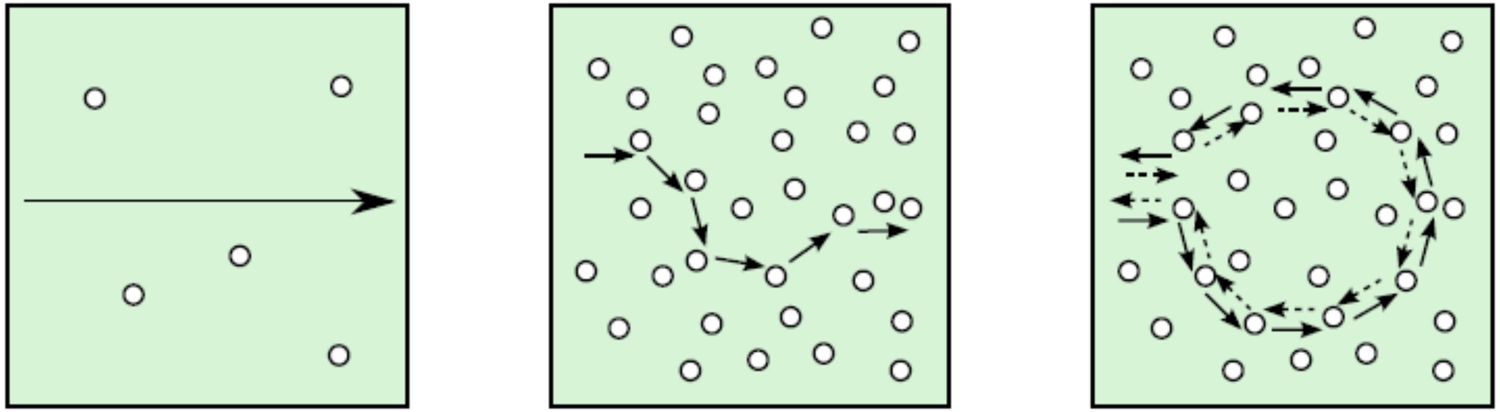
Fudan, Oct 8, 2019

Outline

- ☐ Basics of Weak (Anti-) Localization
  - ✓ Symmetry classes
  - ✓ Berry phase
  - ✓ Interaction effect
  - ✓ Intervalley scattering
  - ✓ Feynman diagrams
- ☐ Topological insulator
- ☐ 2D Material
- ☐ 3D Topological semimetal

HZL & Shun-Qing Shen,
Chinese Physics B 25, 117202 (2016);
Frontiers of Physics 12, 127201 (2017)

Weak (anti-)localization



Ballistic Diffusive Quantum Diffusive

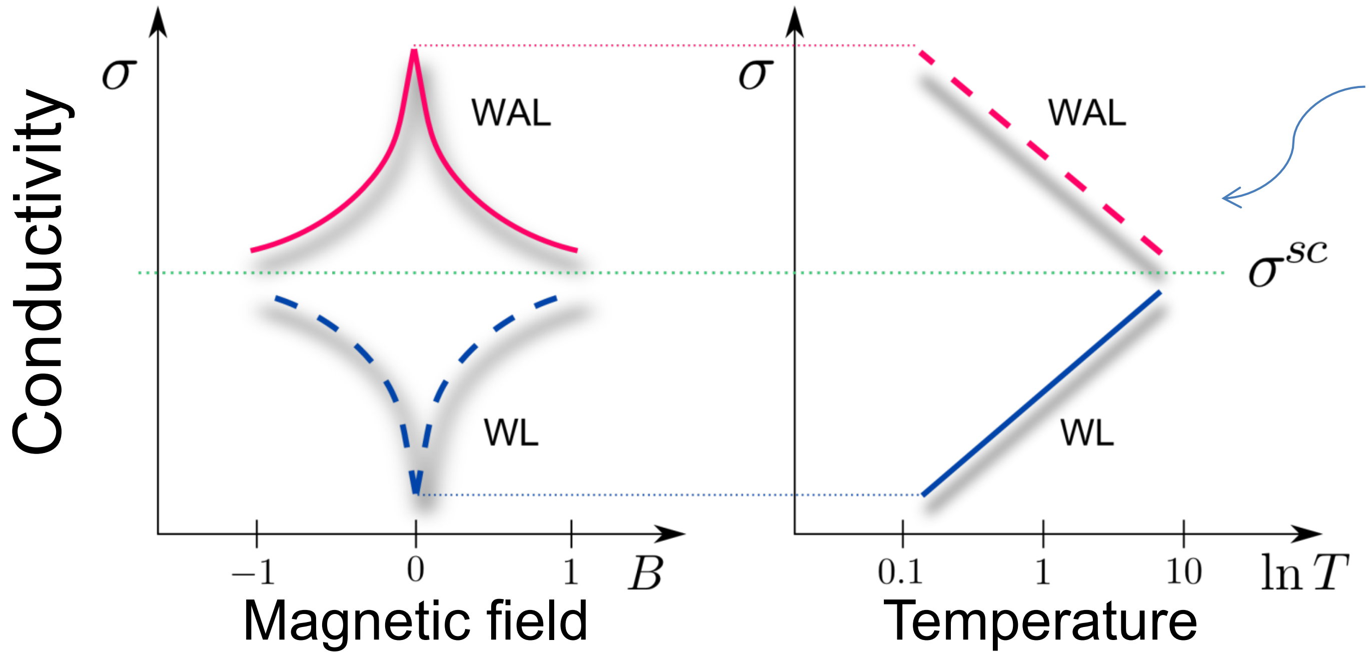
$$\ell \ll \ell_\phi$$

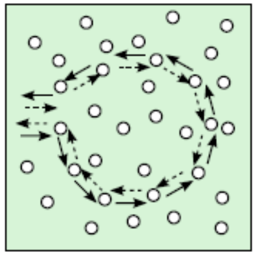
Mean free path

Phase coherence length

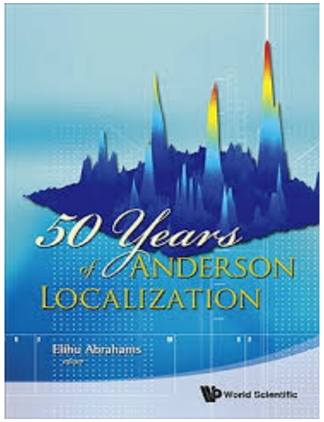
HZL & Shun-Qing Shen,
Chinese Physics B 25, 117202 (2016);
Frontiers of Physics 12, 127201 (2017)

Weak (anti-)localization





Quantum interference correction to semiclassical conductivity σ^{sc}

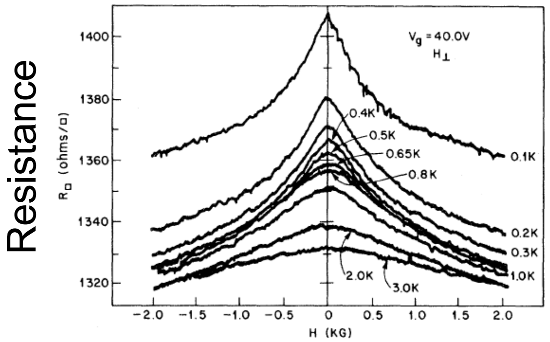


P. W. Anderson, PR 109, 1492 (1958)

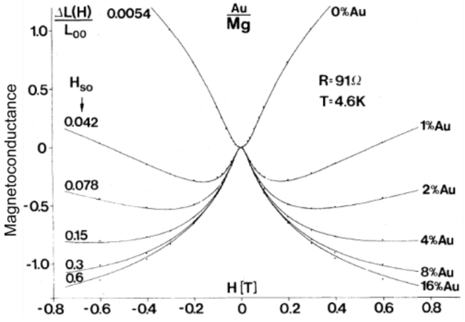
Weak Localization
=
Precursor of Anderson localization

HZL & Shun-Qing Shen,
Chinese Physics B 25, 117202 (2016);
Frontiers of Physics 12, 127201 (2017)

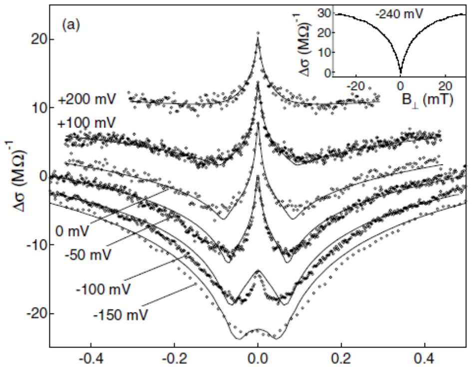
WL/WAL in 2D systems



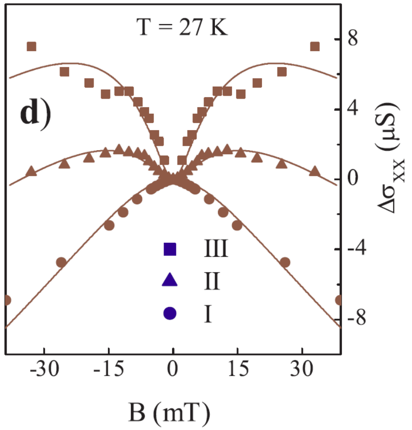
Si-MOSFET
Rosenbaum et al, PRL 47, 1758 (1981); PRL 46, 568 (1981);
Bishop, Dynes, & Tsui, PRB 26, 773 (1982).



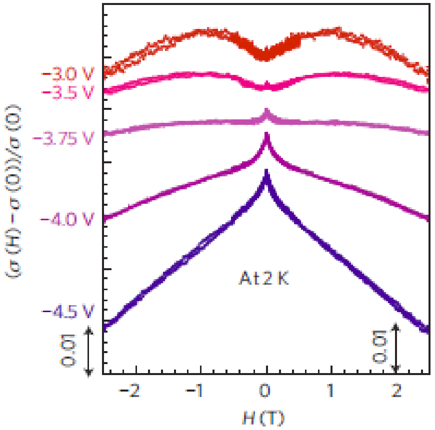
Au-doped Mg thin films
Bergmann, PRL 48, 1046 (1982).



GaAs/GaAlAs
Miller et al, PRL 90, 076807 (2003);
Neumaier et al, PRL 99, 116803 (2007).



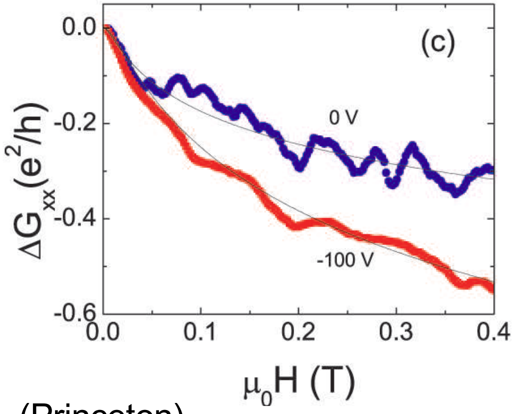
Graphene
Wu, de Heer et al, PRL 98, 136801 (2007); Tikhonenko, et al, PRL 103, 226801 (2009)



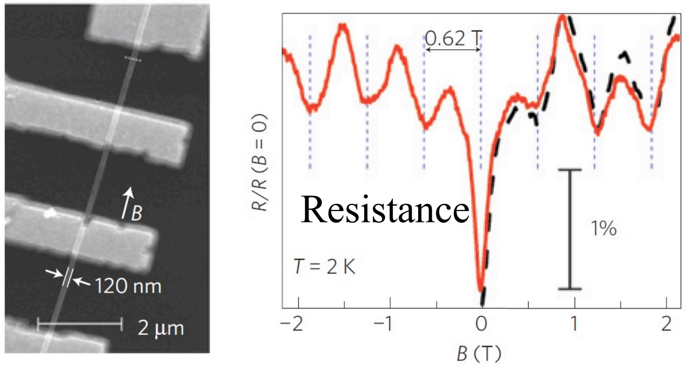
MoS_2, WSe_2
Iwasa et al, Science 2012; Nat. Phys. 2013 (Tokyo);
Peide Ye et al, ACS Nano 2013 (Purdue);
HZL, Di Xiao, Wang Yao & SQ Shen, PRL, 110, 016806 (2013);

WTe2
E. Zhang, Faxian Xiu, Nano Lett. 17, 878 (2017);
InSe
J. Zeng, Feng Miao, et al., PRB 98, 125414 (2018)

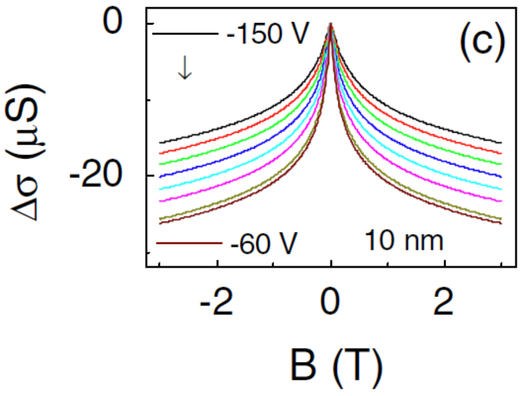
WAL in topological insulators



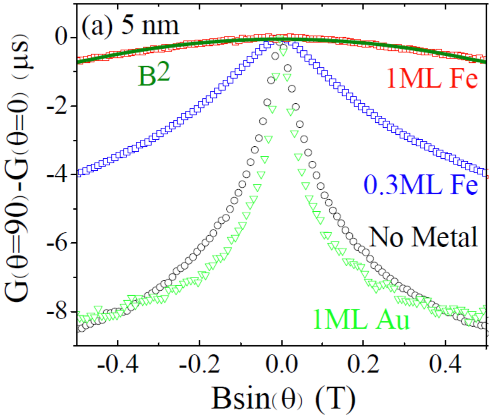
Bi_2Se_3 (Princeton)
Checkelsky, Ong, et al
PRL 103, 246601 (2009); PRL 106, 196801 (2011)



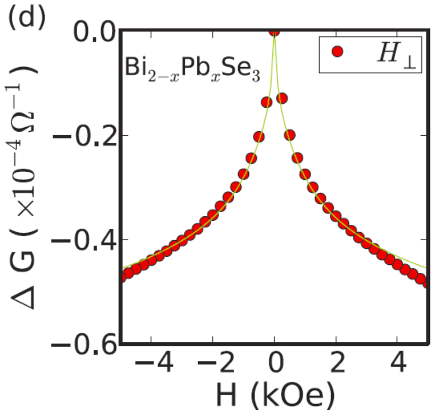
Bi_2Se_3 (Stanford)
H.L. Peng, Yi Cui et al , Nat. Mat. 9, 225 (2009)



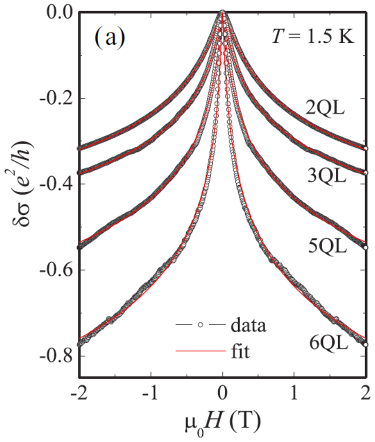
Bi_2Se_3 (IOP)
Chen, Wu, Li, Lu et al,
PRL 105, 176602
(2010)



Bi_2Te_3 (HKUST&HKU)
He, JN Wang, et al,
PRL 106, 166805
(2011)

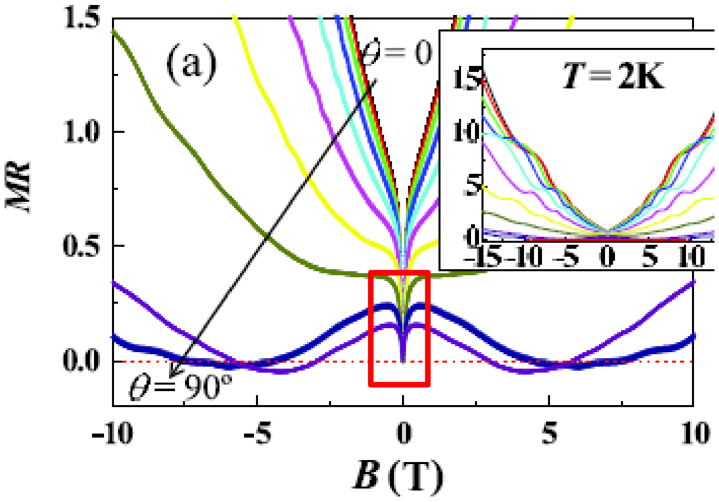


Bi_2Se_3 (PKU, Penn State)
Wang, Moses Chan, et
al, PRB 83, 245438
(2010)

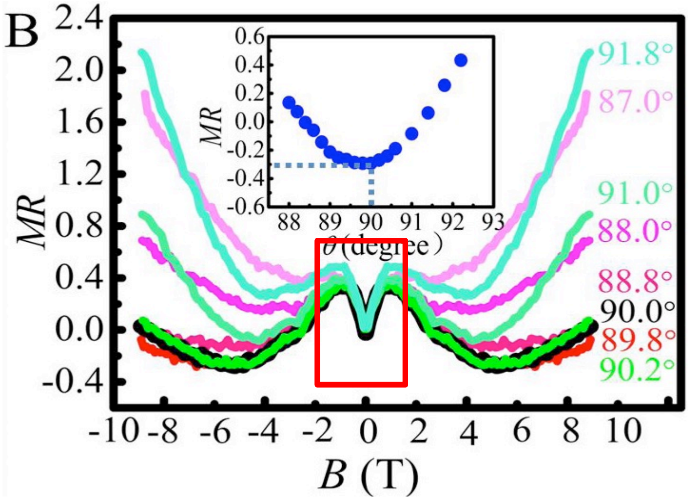


Bi_2Se_3 (Tsinghua, IOP)
Liu, Yayu Wang, et
al, PRB 83, 165440
(2011); PRL 108,
036805 (2012)

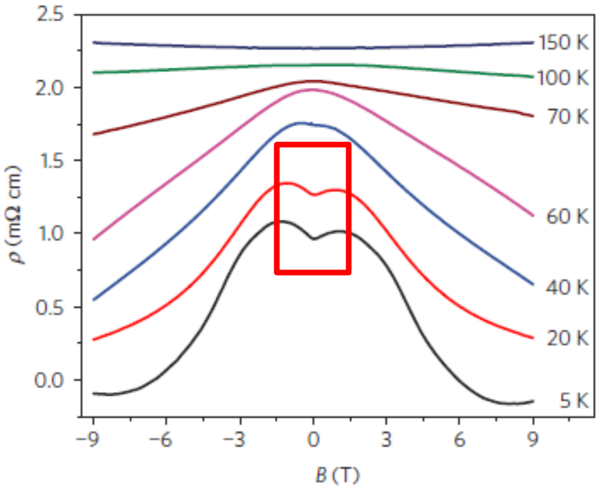
WAL in topological semimetals



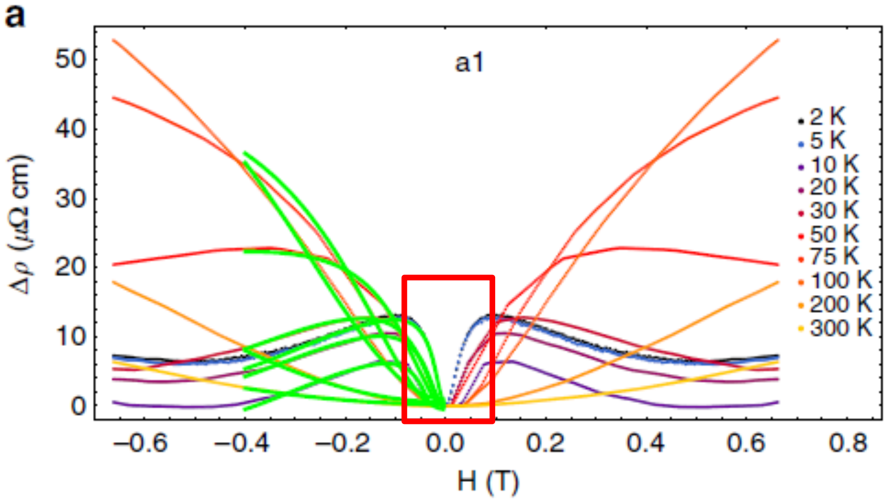
BiSb alloy, Kim et al, PRL 111, 246603 (2013)



TaAs, Huang et al, PRX 5, 031023 (2015)



ZrTe5, Li et al, Nat. Phys. 12, 550 (2016)

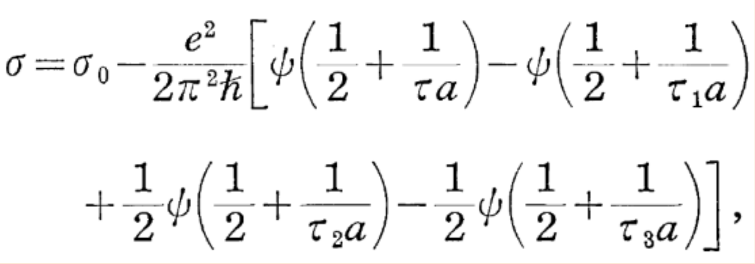


TaAs, Zhang et al, Nat. Comm. 7, 10735 (2016)

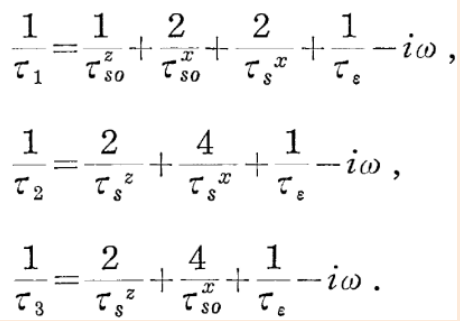
WL or WAL? Symmetry classes

Impurity scattering	Symmetry	Time-reversal	Spin-rotational	Transport
Scalar	Orthogonal	✓	✓	WL
Spin-orbit	Symplectic	✓	✗	WAL
Magnetic	Unitary	✗		Semiclassical

Symmetry classes of random ensembles [F. J. Dyson, J. Math. Phys. 1962]



$$\sigma=\sigma_0-\frac{e^2}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2}+\frac{1}{\tau a}\right)-\psi\left(\frac{1}{2}+\frac{1}{\tau_1 a}\right)+\frac{1}{2}\psi\left(\frac{1}{2}+\frac{1}{\tau_2 a}\right)-\frac{1}{2}\psi\left(\frac{1}{2}+\frac{1}{\tau_3 a}\right)\right],$$



$$\frac{1}{\tau_1}=\frac{1}{\tau_{so}^z}+\frac{2}{\tau_{so}^x}+\frac{2}{\tau_s^x}+\frac{1}{\tau_\varepsilon}-i\omega\,,$$

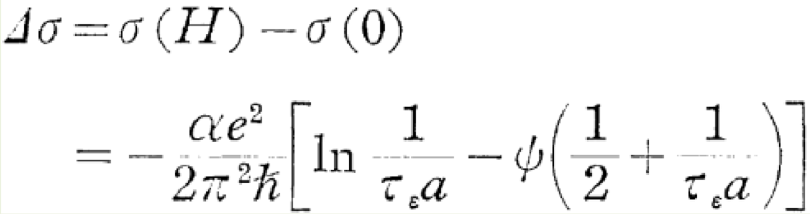
$$\frac{1}{\tau_2}=\frac{2}{\tau_s^z}+\frac{4}{\tau_s^x}+\frac{1}{\tau_\varepsilon}-i\omega\,,$$

$$\frac{1}{\tau_3}=\frac{2}{\tau_s^z}+\frac{4}{\tau_{so}^x}+\frac{1}{\tau_\varepsilon}-i\omega\,.$$

τ_{so} Spin-orbit scattering

τ_{s} Magnetic scattering

The famous HLN formula



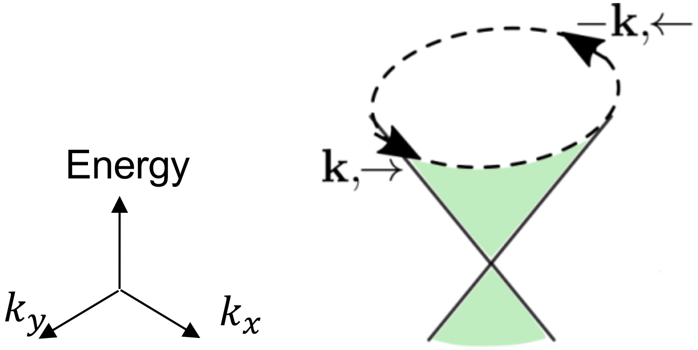
$$\Delta\sigma=\sigma(H)-\sigma(0)$$
$$=-\frac{\alpha e^2}{2\pi^2\hbar}\left[\ln\frac{1}{\tau_\varepsilon a}-\psi\left(\frac{1}{2}+\frac{1}{\tau_\varepsilon a}\right)\right]$$

- ☐ No spin-orbit scattering $\alpha=1$
- ☐ Strong spin-orbit scattering $\alpha=-\frac{1}{2}$

Hikami, Larkin, Nagaoka, Prog. Theor. Phys. 63 7071(1980)
Altshuler, Aronov, Larkin, Khmel’nitsikii, Sov. Phys. JETP 54, 411 (1981)

WL or WAL? Berry phase

Massless Dirac fermion



$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

Spin-momentum locking

Spinor wave function $\psi_k = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -ie^{i\phi} \end{pmatrix}$

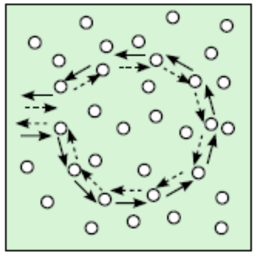
Momentum angle $\tan\phi \equiv \frac{k_y}{k_x}$

Berry phase [Kun Huang & Born 1954;
Berry PRLSA 392, 45 (1984)]

$$\begin{aligned}
\varphi &\equiv -i\int_0^{2\pi} d\phi \langle\psi_k|\frac{\partial}{\partial\phi}|\psi_k\rangle \\
&= -\frac{i}{2}\int_0^{2\pi} d\phi (1, ie^{-i\phi})\frac{\partial}{\partial\phi}\begin{pmatrix} 1 \\ -ie^{i\phi} \end{pmatrix} \\
&= -\frac{i}{2}\int_0^{2\pi} d\phi (1, ie^{-i\phi})\begin{pmatrix} 0 \\ e^{i\phi} \end{pmatrix} \\
&= \frac{1}{2}\int_0^{2\pi} d\phi \\
&= \pi
\end{aligned}$$

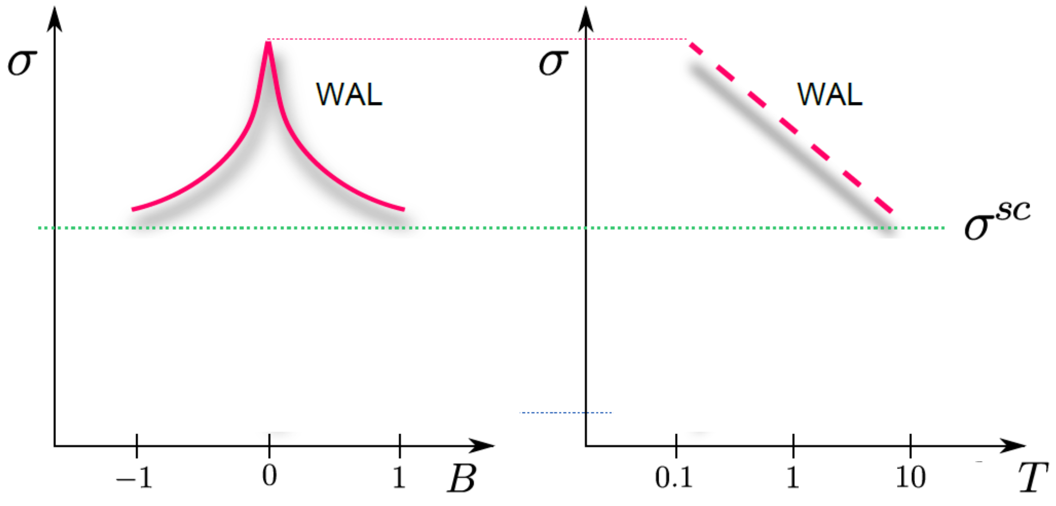
⇓

Destructive quantum interference = WAL

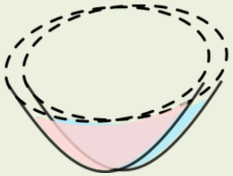


Suzuura & Ando, PRL 89, 266603 (2002)

WAL: Symmetry class vs. Berry phase



Conventional electrons



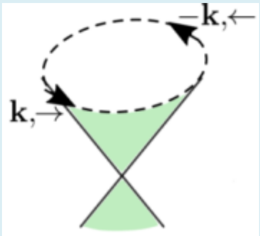
$$H = \frac{p^2}{2m} + \sum_i \vec{\sigma}\cdot\nabla u(r-R_i)\times\vec{p}$$

Spin-orbit scattering →

[Hikami, Larkin & Nagaoka,
Prog. Theo. Phys. 1980]

Symplectic
symmetry

Massless Dirac fermions



$$H = v\vec{\sigma}\times\vec{p} + \sum_i \sigma_0 u(r-R_i)$$

← Spin-momentum locking

π Berry phase

How to calculate? Feynman diagrams

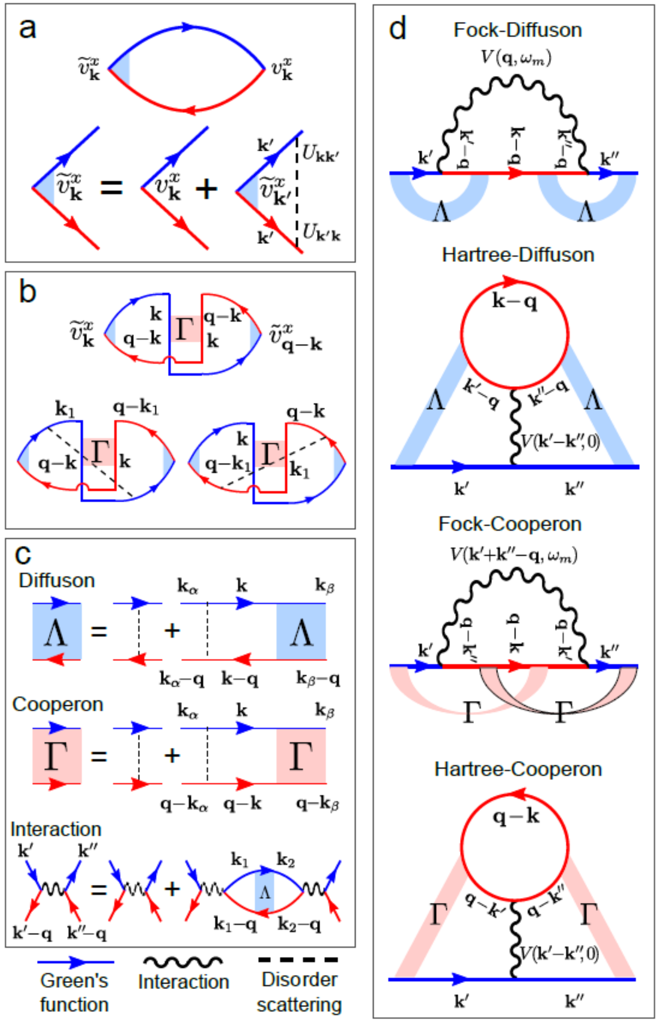
Total conductivity $\sigma = \sigma^{sc} + \sigma^{qi} + \sigma^{ee}$

σ^{sc} **Drude**

Ladder diagram correction to velocity [Shon & Ando, JPSJ 67, 2421 (1998)]

σ^{qi} **WL / WAL**

Maximally crossed diagram
Langer and Neal,
PRL 16, 984 (1966)
The dressed Hikami boxes
McCann, Kechedzhi, Fal’ko,
Suzuura, Ando, and Al'tshuler,
PRL 97, 146805 (2006)



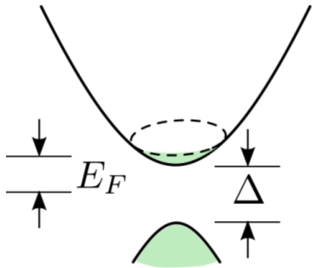
AAF effect σ^{ee}

Altshuler, Aronov, & Lee,
Phys. Rev. Lett. 44, 1288 (1980)
Fukuyama, JPSJ 48, 2169 (1980)

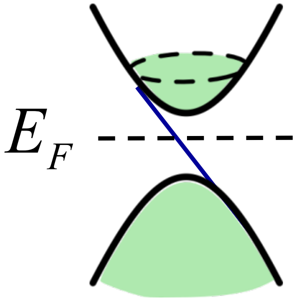
Reviews
Bergmann, Phys. Rep. 107, 1 (1984); Altshuler and Aronov; Fukuyama, in “Electron-electron interactions in disordered systems” (North-Holland, Amsterdam, 1985)

WL or WAL? Berry phase

2D massive Dirac system is important because it may support Quantized anomalous Hall effect (QAHE)

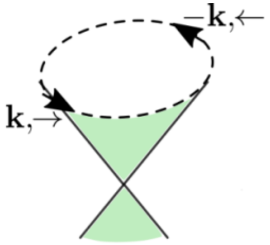


Large-gap



$$\varphi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right) = \begin{cases} 0/2\pi, & \dfrac{\Delta}{2E_F} = 1 \quad \text{WL} \\ \pi, & \dfrac{\Delta}{2E_F} = 0 \quad \text{WAL} \end{cases}$$

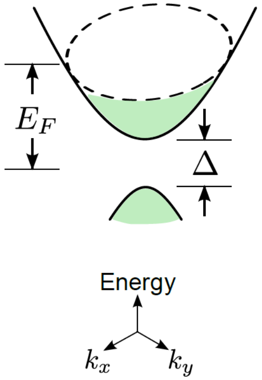
Berry phase

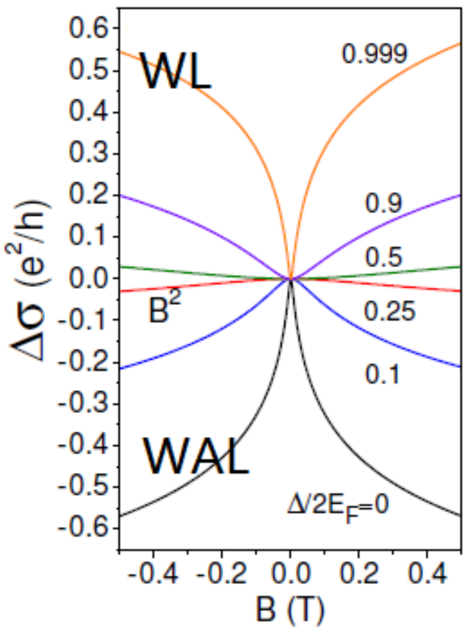


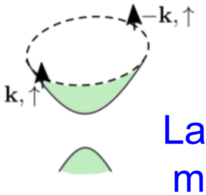
gapless

HZL, Junren Shi & Shun-Qing Shen, PRL, 107 076801 (2011)

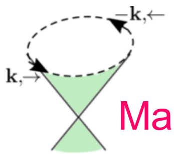
WAL-WL crossover







Large-mass



Massless

Berry phase

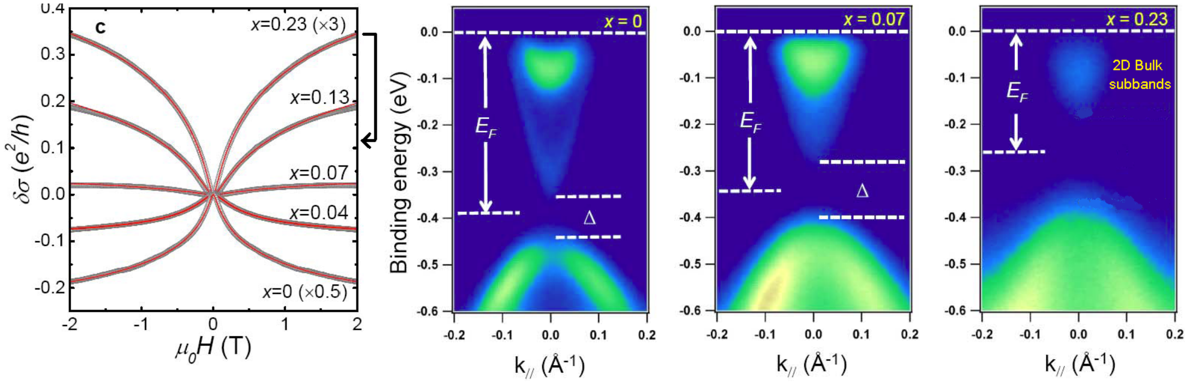
$$\varphi = \pi(1 \pm \frac{\Delta}{2E_F})$$

$$= \begin{cases} 0/2\pi, & \frac{\Delta}{2E_F} = 1 \\ \pi, & \frac{\Delta}{2E_F} = 0 \end{cases}$$

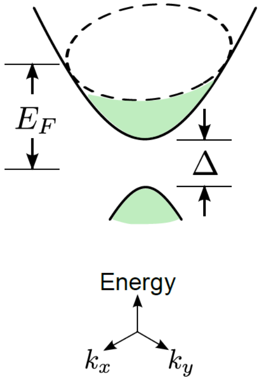
HZL, Junren Shi & Shun-Qing Shen, PRL, 107 076801 (2011)

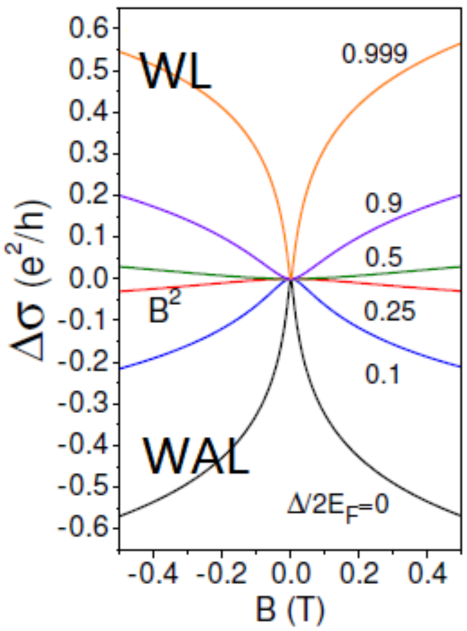
Observed by many groups, driven by the search for QAHE

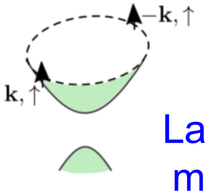
Liu, Yayu Wang et al, PRL 108 036805 (2012) (Tsinghua & IOP, China)
3QL $Bi_{2-x}Cr_xSe_3$



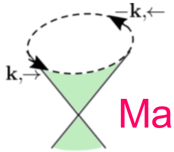
WAL-WL crossover







Large-mass



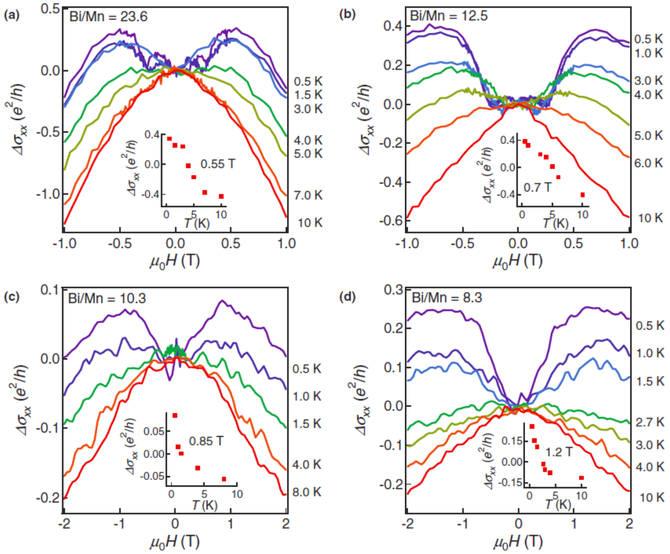
Massless

Berry phase

$$\varphi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right)$$

$$= \begin{cases} 0/2\pi, & \dfrac{\Delta}{2E_F} = 1 \\ \pi, & \dfrac{\Delta}{2E_F} = 0 \end{cases}$$

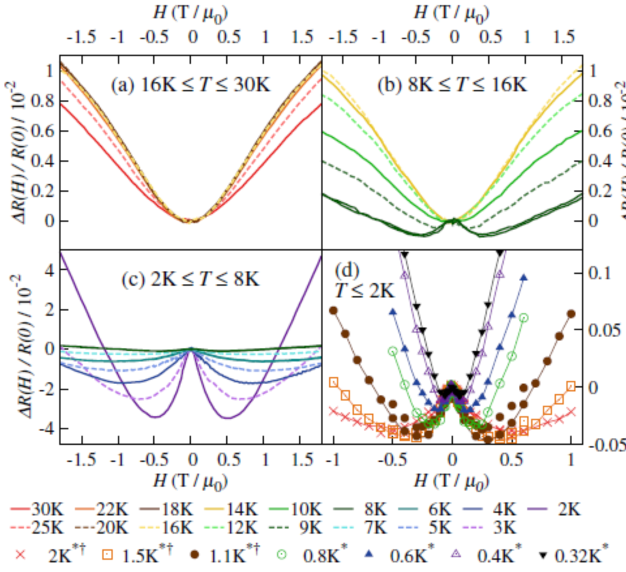
HZL, Junren Shi & Shun-Qing Shen, PRL, 107 076801 (2011)



Zhang, Samarth et al, (Penn State) PRB 86, 205127 (2012)
Editors' suggestion

Mn-Bi2Se3

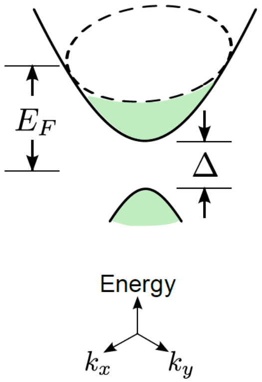
WAL-WL crossover as a signature of ferromagnetic phase transition with Tc~5K

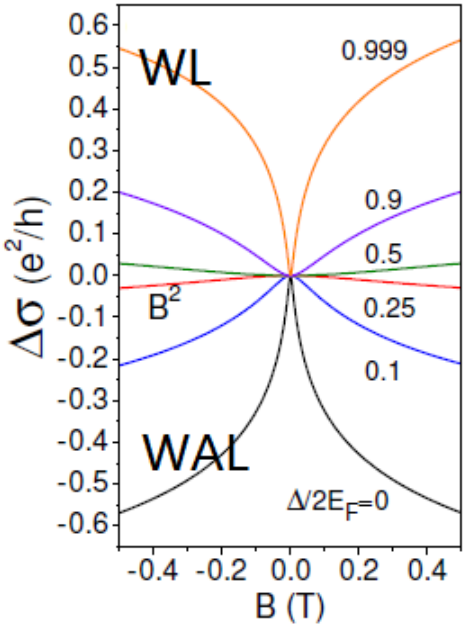


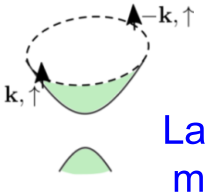
Yang, Kapitulnik et al (Stanford)
EuS/Bi2Se3
Europium Sulfide
PRB 88, 081407(R) (2013)
Editors' suggestion

Also:
EuS/Bi2Se3
Wei, Moodera, et al (MIT) 2013

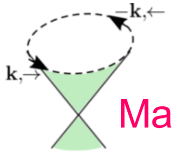
WAL-WL crossover







Large-mass



Massless

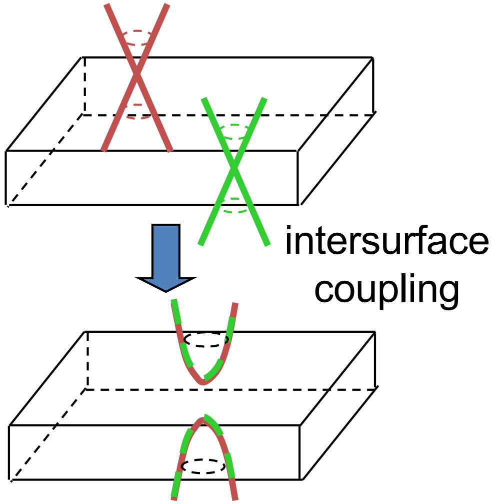
Berry phase

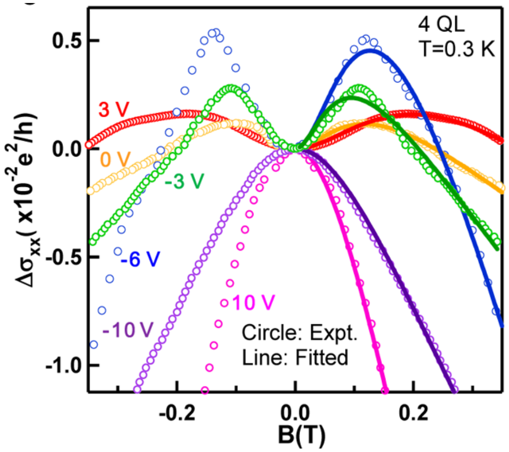
$$\varphi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right) = \begin{cases} 0/2\pi, & \dfrac{\Delta}{2E_F} = 1 \\ \pi, & \dfrac{\Delta}{2E_F} = 0 \end{cases}$$

HZL, Junren Shi & Shun-Qing Shen, PRL, 107 076801 (2011)

Finite size effect

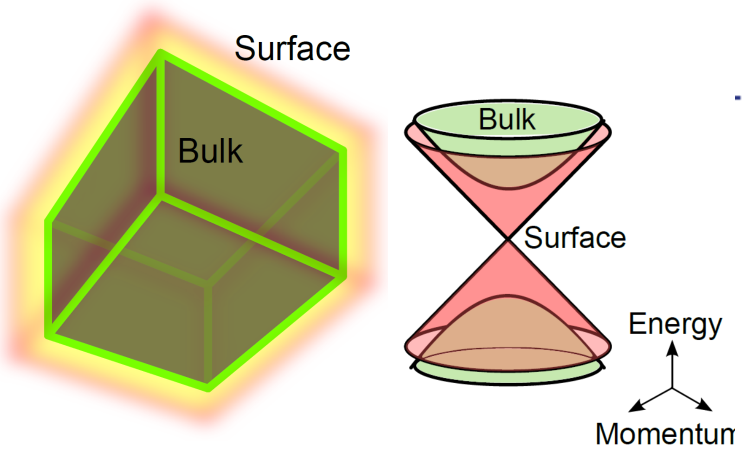
Lu, Shan, Yao, Niu & Shen PRB 81, 115407 (2010); Liu, Zhang et al, PRB(R) 2010; Linder et al, PRB 2009.

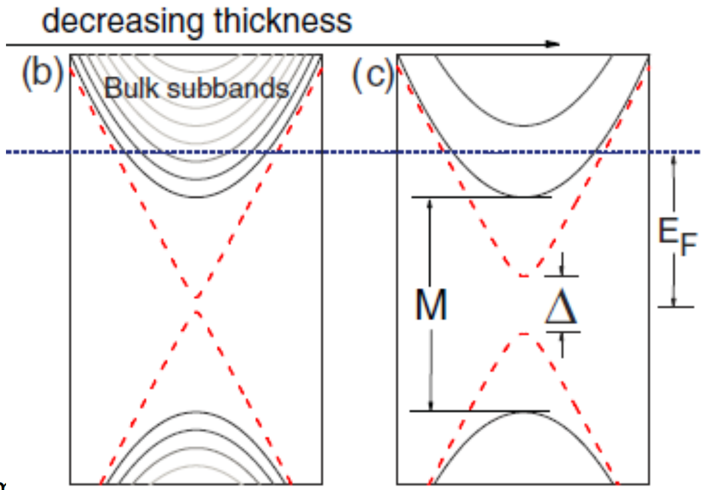




Lang, K. L, Wang (UCLA)
Nano Lett. (2013)
4 QL $Bi_{1.14}Sb_{0.86}Te_3$

WL of TI bulk states





Berry phase

$$\varphi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right) = \begin{cases} 0/2\pi, & \frac{\Delta}{2E_F} = 1 \quad \text{WL} \\ \pi, & \frac{\Delta}{2E_F} = 0 \end{cases}$$

HZL & Shun-Qing Shen, PRB 84, 125138 (2011)

Bulk states have strong spin-orbit coupling but also have WL, instead of WAL

Later, supported by

- ✓ Garate & Glazman, PRB, 86, 035422 (2012);
- ✓ Zhang, Kapitunik, et al, PRB 2013 “Weak localization effects as evidence for bulk quantization in Bi2Se3 thin films” …

Topological semimetal: Berry phase & Monopole

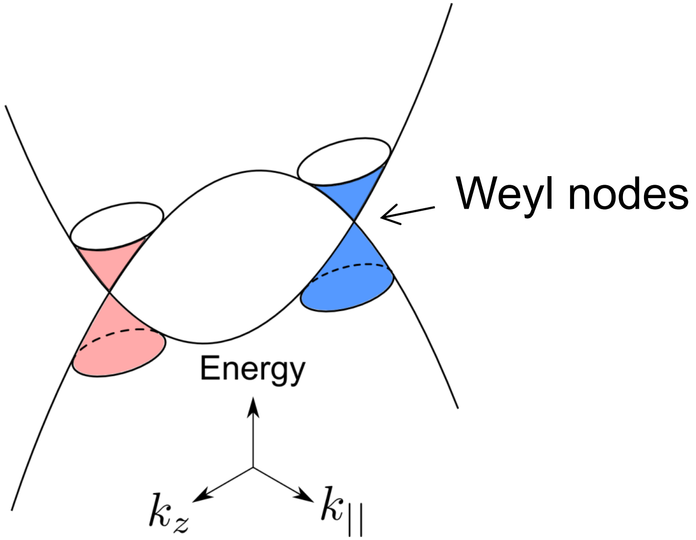
Hamiltonian $H = v k \cdot \sigma$

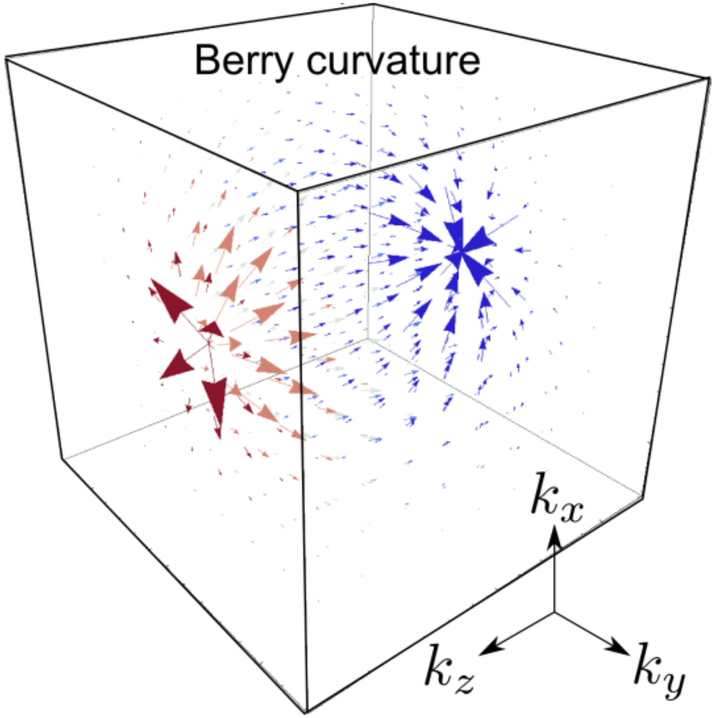
Eigen states $$|u_+\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} e^{i\varphi} \end{pmatrix}$$

Berry connection $$\mathbf{A} \equiv -i\langle u_+ |\nabla_{\mathbf{k}}| u_+\rangle$$

Berry curvature $$\begin{aligned} \mathbf{\Omega} &\equiv \nabla \times \mathbf{A} \\ &= \mp \frac{1}{2k^2}\hat{e}_k \end{aligned}$$

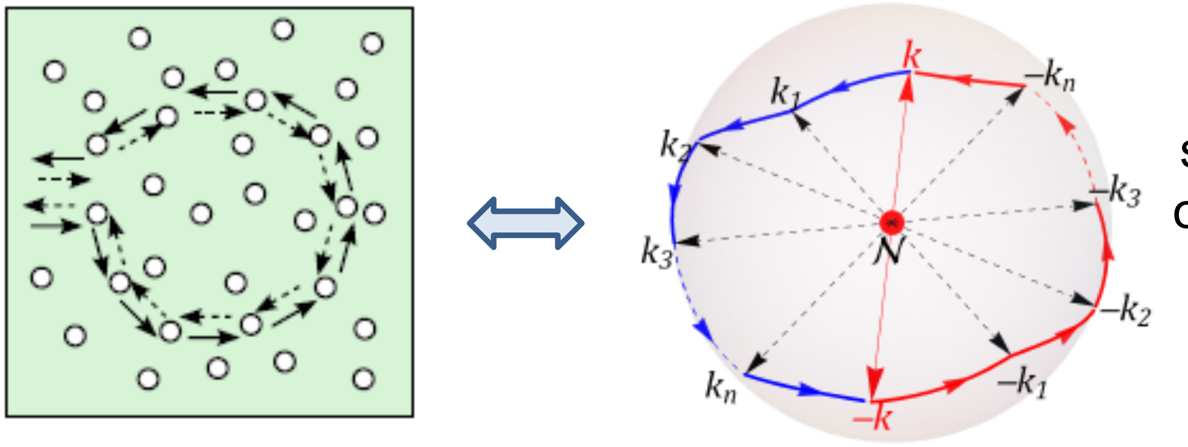
Topological charge $$\begin{aligned} \mathcal{N} &= \frac{1}{2\pi}\int_{\Sigma} d\mathbf{S}\cdot\mathbf{\Omega} \\ &= \mp 1 \end{aligned}$$





Lu & Shen, Front. Phys. 12, 127201 (2017)

Topological semimetal: Berry phase & Monopole



Time-reversed
scattering loops
on Fermi sphere
=
Great circle

Berry phase

$$\gamma = \oint_C d\ell \cdot A = \frac{1}{2}\oint_\Sigma dS \cdot \Omega = \pi N$$

Monopole charge

Great circle

Berry connection

Fermi sphere

Berry curvature

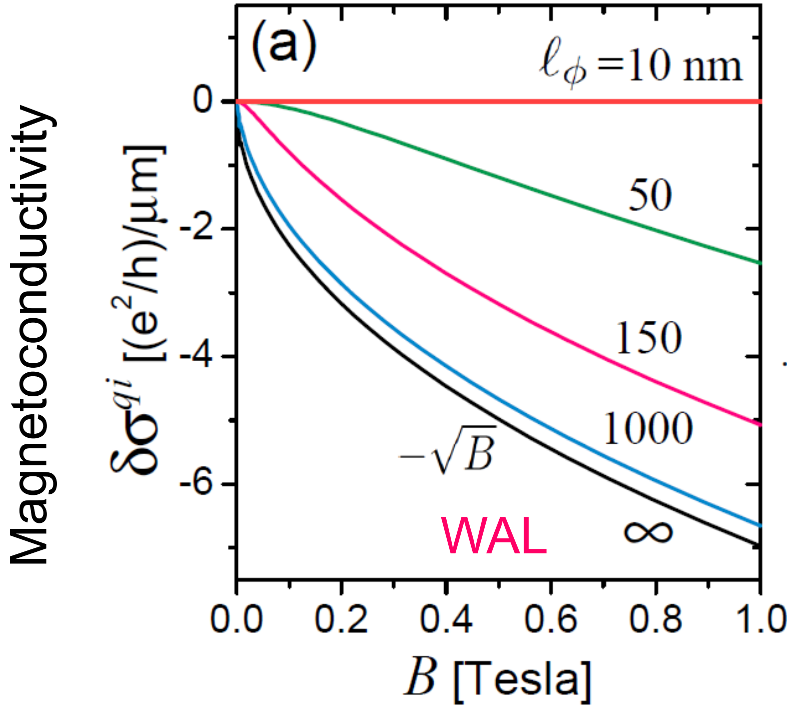
$$N = \begin{cases} 1, & \text{WAL} \quad \text{Weyl semimetal} \\ 2, & \text{WL} \quad \text{double-Weyl semimetal} \end{cases}$$

HgCr2Se4 Xu, Weng, Wang, Dai, & Fang, PRL 107, 186806 (2011)

SrSi2 S.-M. Huang, et al., PNAS 113, 1180 (2016)

Xin Dai, HZL, Shun-Qing Shen, Hong Yao, PRB 92,035203(R)(2016)

3D WAL magnetoconductivity



Phase coherence length ℓ_ϕ >> ℓ =10 nm Mean Free path

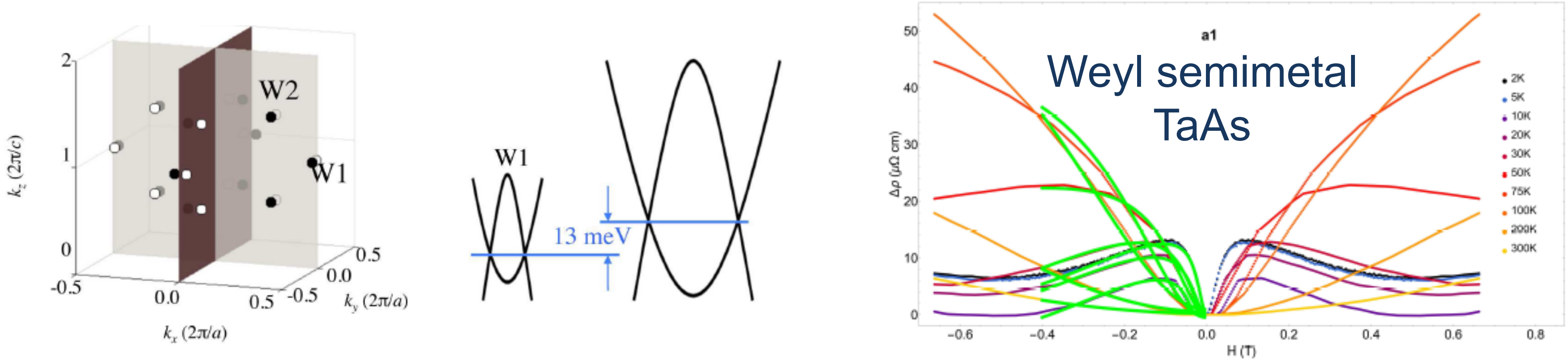
(1) *B* along arbitrary direction

(2) $\delta\sigma = \begin{cases} \sim B^2, & B \ll B_C \\ \sim\sqrt{B}, & B \gg B_C \end{cases}$

Fitting formula $$\sigma_{\mathrm{xx}} = 8C_{\mathrm{W}}B^2 - \boxed{C_{\mathrm{WAL}}\left(\sqrt{B}\frac{B^2}{{B_C}^2+B^2} + \gamma B^2\frac{{B_C}^2}{{B_C}^2+B^2}\right)} + \sigma_0$$

HZL & Shun-Qing Shen, PRB 92,035203(2015)

3D WAL magnetoconductivity



Zhang et al, Nature Comm. 7, 10735 (2016), Shuang Jia @PKU & Hasan @Princeton

Also, used by Wang, Liu, Baigeng Wang, Xiangang Wan, Feng Miao @NJU, Nature Comm. 7, 13142 (2016) “Gate-tunable negative longitudinal magnetoresistance in the predicted type-II Weyl semimetal WTe2”.

Fitting formula

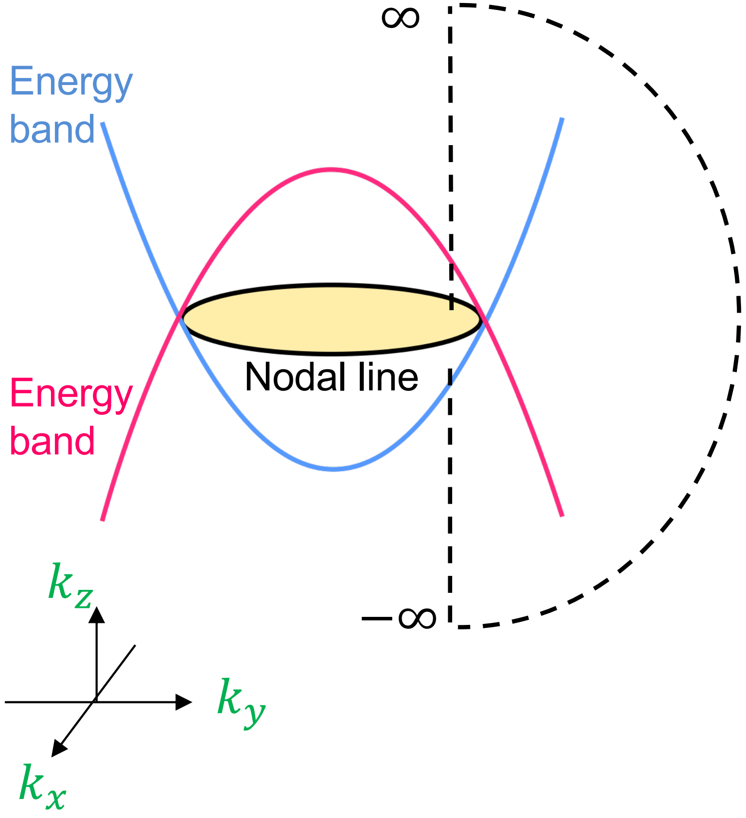
$$\sigma_{xx} = 8C_W B^2 - \boxed{C_{WAL}\left(\sqrt{B}\frac{B^2}{{B_c}^2 + B^2} + \gamma B^2 \frac{{B_c}^2}{{B_c}^2 + B^2}\right)} + \sigma_0$$

HZL & Shun-Qing Shen, PRB 92,035203(2015)

WL-WAL in nodal-line semimetals

$$H = M_k\sigma_z + Ak_z\sigma_x$$
$$M_k = m_0 - m_1\left(k_x^2 + k_y^2 + k_z^2\right)$$

Nodal-line semimetal, if $m_0 m_1 > 0$



Eigen state (k_x, k_y as parameters)

$$|\psi\rangle = \begin{pmatrix} \mathrm{sgn}(k_z)\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}, \quad \cos\theta \equiv \frac{M_k}{\sqrt{M_k^2+(Ak_z)^2}}$$

Winding number

$$n(k_x, k_y) = \frac{1}{\pi}\int_{-\infty}^{\infty} dk_z \langle\psi|i\partial_{k_z}|\psi\rangle$$
$$= \frac{1}{2}\{\mathrm{sgn}[m_0 - m_1(k_x^2 + k_y^2)] + \mathrm{sgn}(m_1)\}$$
$$= \begin{cases} 1, & \text{if } k_x^2 + k_y^2 < \dfrac{m_0}{m_1}; \\ 0, & \text{if } k_x^2 + k_y^2 > \dfrac{m_0}{m_1}. \end{cases}$$

Drumhead Surface states

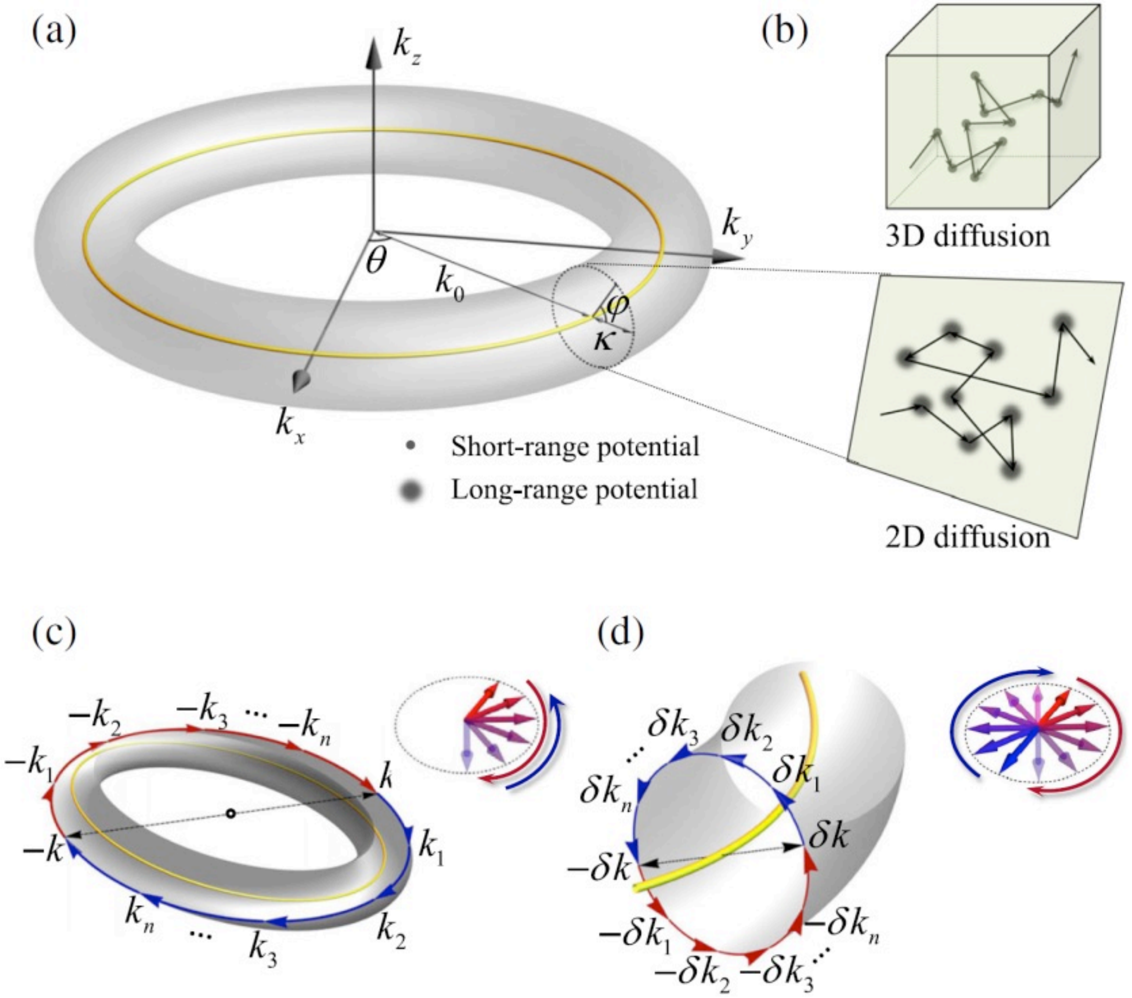
➡ $(-\infty = \infty)$

Berry phase

$$\varphi \equiv -i\int_0^{2\pi} d\phi \langle\psi_k|\frac{\partial}{\partial\phi}|\psi_k\rangle$$
$$= \pi$$

Cequn Li , C. M. Wang, Bo Wan, Xiangang Wan, HZL, and X. C. Xie, PRL 120, 146602 (2018)

WL-WAL in nodal-line semimetals



❑ Long-range impurity potential
Scattering around the nodal ring
π Berry phase
Symplectic class
2D weak antilocalization

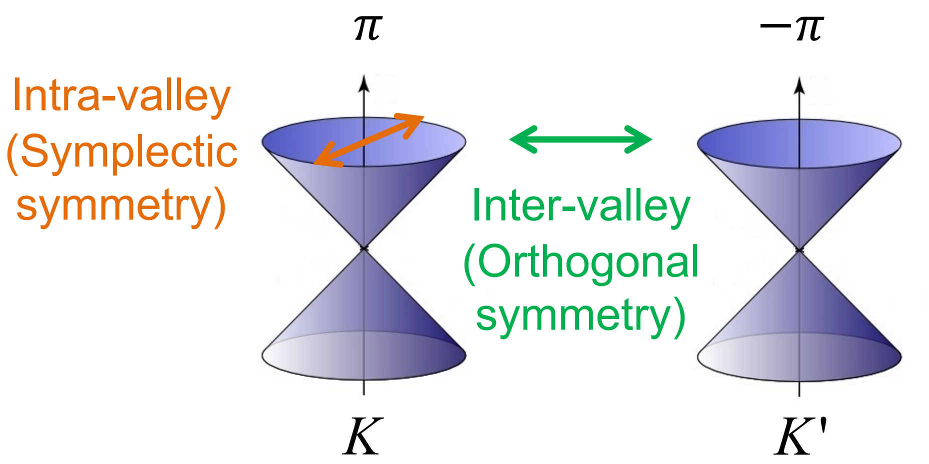
$$\delta\sigma \sim \ln(B)$$

❑ Short-range impurity potential
Scattering along the nodal ring
no Berry phase
Orthogonal class
3D weak localization effect

$$\delta\sigma \sim \sqrt{B}$$

Wei Chen, HZL, Oded Zilberberg, PRL 122, 196603 (2019)

Intervalley scattering



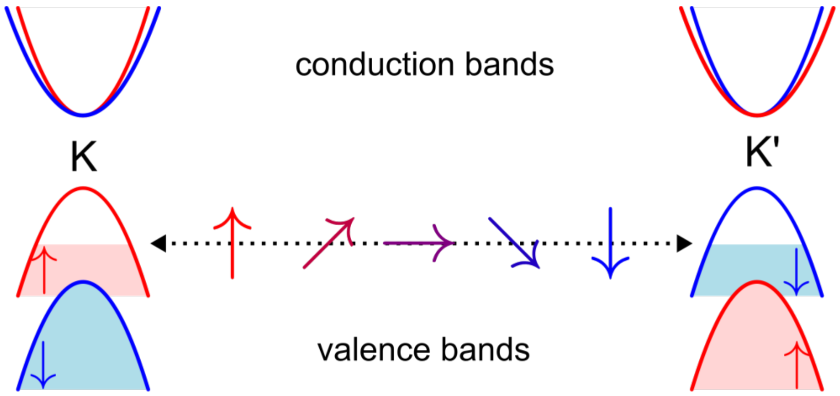
2D Graphene

Suzuura & Ando, PRL 2002; McCann, Kechedzhi, Fal’ko, Suzuura, Ando, and Al‘tshuler, PRL 2006; Tikhonenko, et al, PRL 103, 226801 (2009)

3D Weyl semimetal

HZL & Shun-Qing Shen, PRB 92, 035203 (2015)

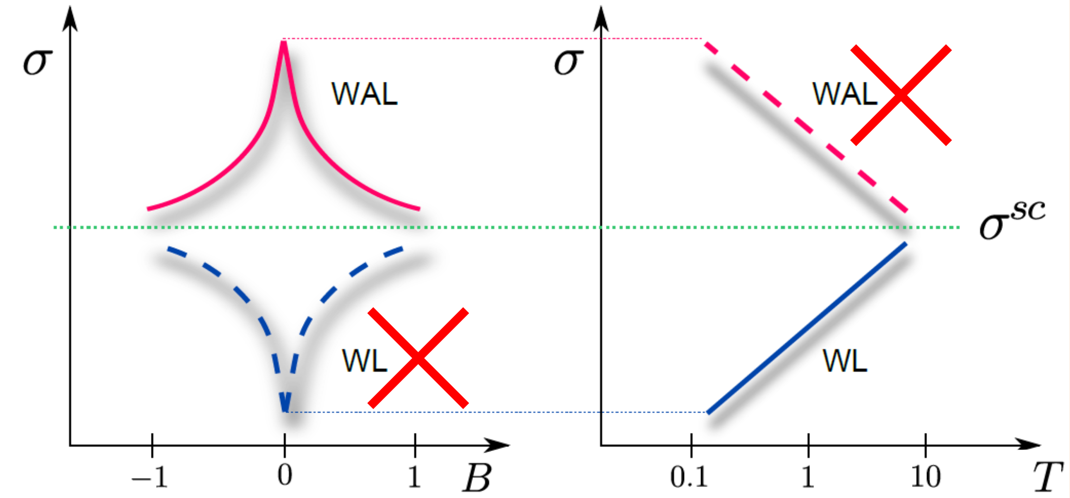
	Intravalley scattering	Intervalley scattering
Graphene / Weyl semimetal	WAL	WL
p-type MoS_2	WL	WAL

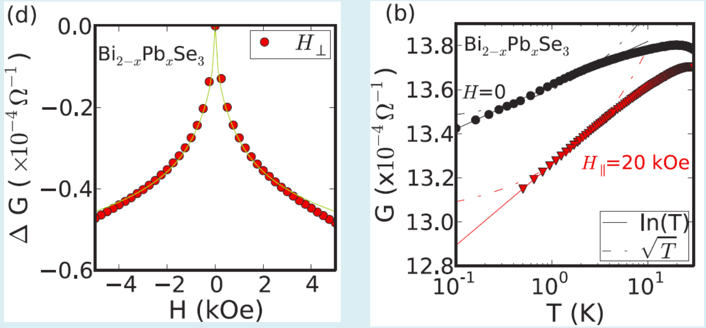


Hole-doped MoS2

HZL, Wang Yao, Di Xiao, & Shun-Qing Shen, PRL 110, 016806 (2013)

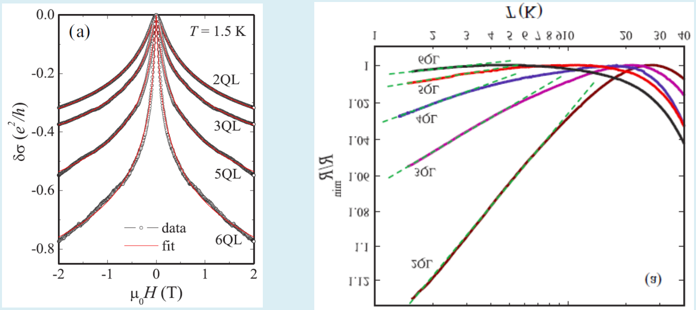
Interaction effect

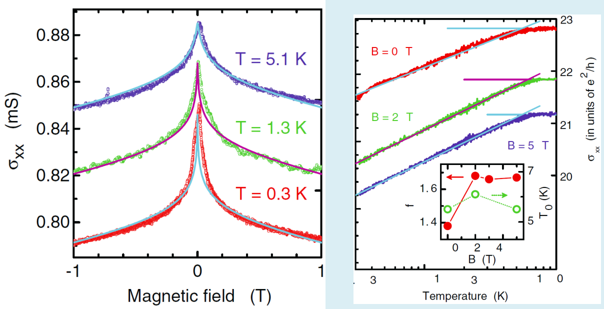




Wang (PKU), Moses Chan (Penn state), et al, PRB 83, 245438 (2010).
Bi2Se3 and Pb-Bi2Se3 (Tsinghua, IOP)

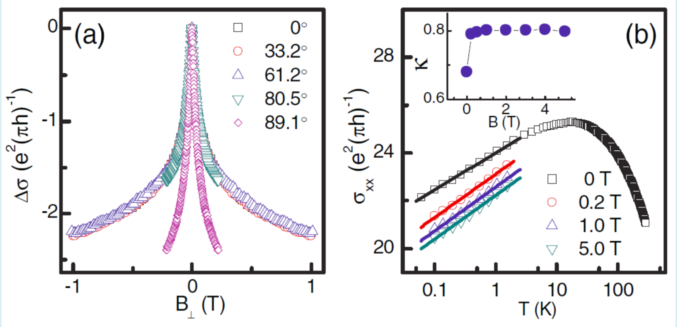
Liu, Yayu Wang (Tsinghua), et al, PRB 83, 165440 (2011)
Bi2Se3 ultrathin films

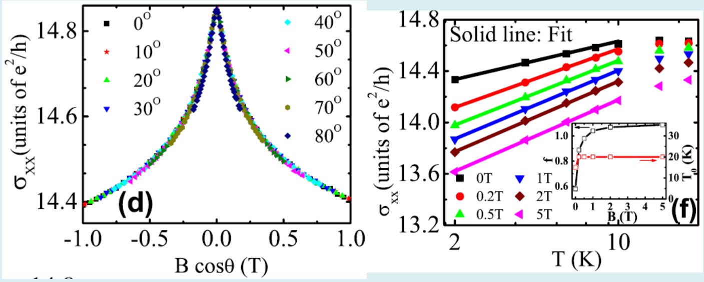




Takagaki et al (Berlin), PRB, 85, 115314 (2012)
Bi2Se3

Chen, Y.Q. Li, K. H. Wu, L. Lu, et al (IOP), PRB, 83, 241304 (R) (2011)
Bi2Se3

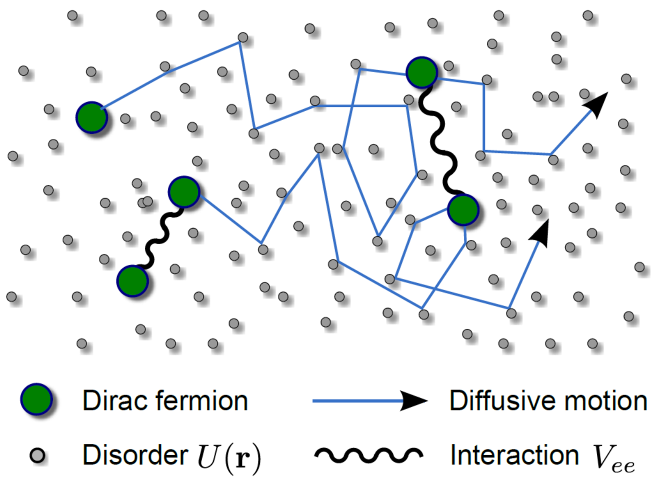


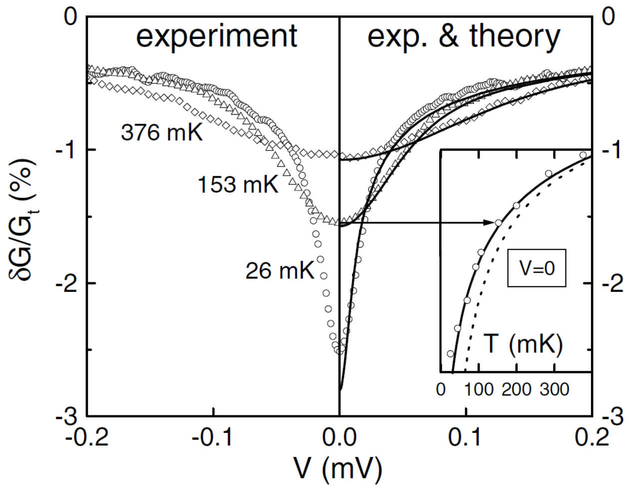


Roy et al (Austin), APL 102, 163118 (2013);
Bi2Te3 ultrathin film

Interaction effect

Altshuler-Aronov-Fukuyama effect





Pierre et al, PRL 86, 1590 (2001).
"Electrodynamic Dip in the Local Density of States of a Metallic Wire".

Electron-electron
interaction +
Disorder scattering

⇩

WL-like temperature dependence of the conductivity

$$\sigma^{ee} \propto \begin{cases} \sqrt{T}, & 3\mathrm{D} \\ \ln T, & 2\mathrm{D} \\ 1/\sqrt{T}, & 1\mathrm{D} \end{cases}$$

Altshuler and Aronov; Fukuyama,
in Electron-electron interactions in disordered systems (North-Holland, Amsterdam, 1985)

Interaction effect

Temperature dependence of conductivity

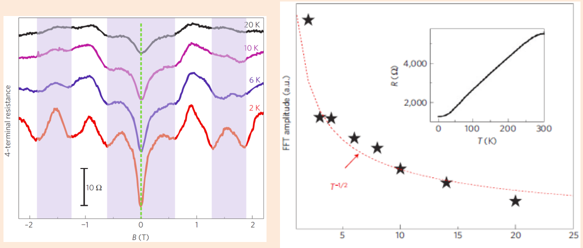
	Altshuler-Aronov-Fukuyama effect	WL	WAL
2D	$\ln(T)$	$\ln(T)$	$-\ln(T)$
3D	$\sqrt{T}$	$T^{p/2}$	$T^{p/2}$

Altshuler et al, J. Phys. C 15, 7367 (1982); Fukuyama, JPSJ, 53, 3299, (1984); Lee & Ramakrishnan, RMP 57, 287 (1985)

p is from $\ell_{\varphi} \sim T^{-p/2}$, depends on dimensionality and decoherence mechanisms. In disordered metals

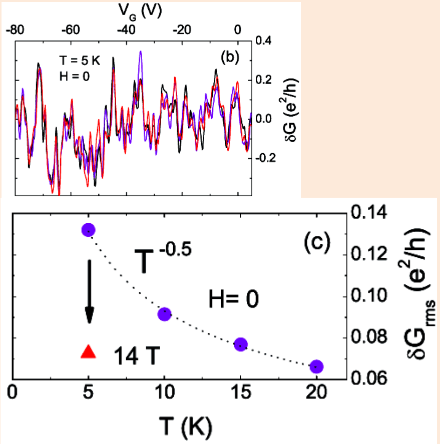
p	EEI or EM	E-ph
1D	2/3	
2D	1	3
3D	3/2	

In topological insulators, $p \sim 1$



AB oscillation

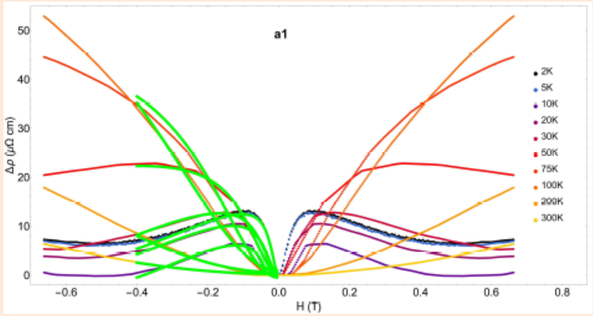
Peng, Yi Cui et al (Stanford), Nat. Mater. 9, 225 (2010).



Universal conductance fluctuation

Checkelsky, Ong et al (Princeton), PRL 106, 196801 (2011)

In topological semimetal TaAs, $p \sim 3/2$

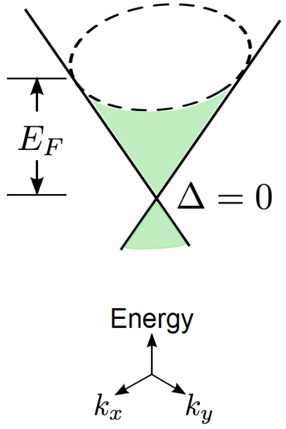


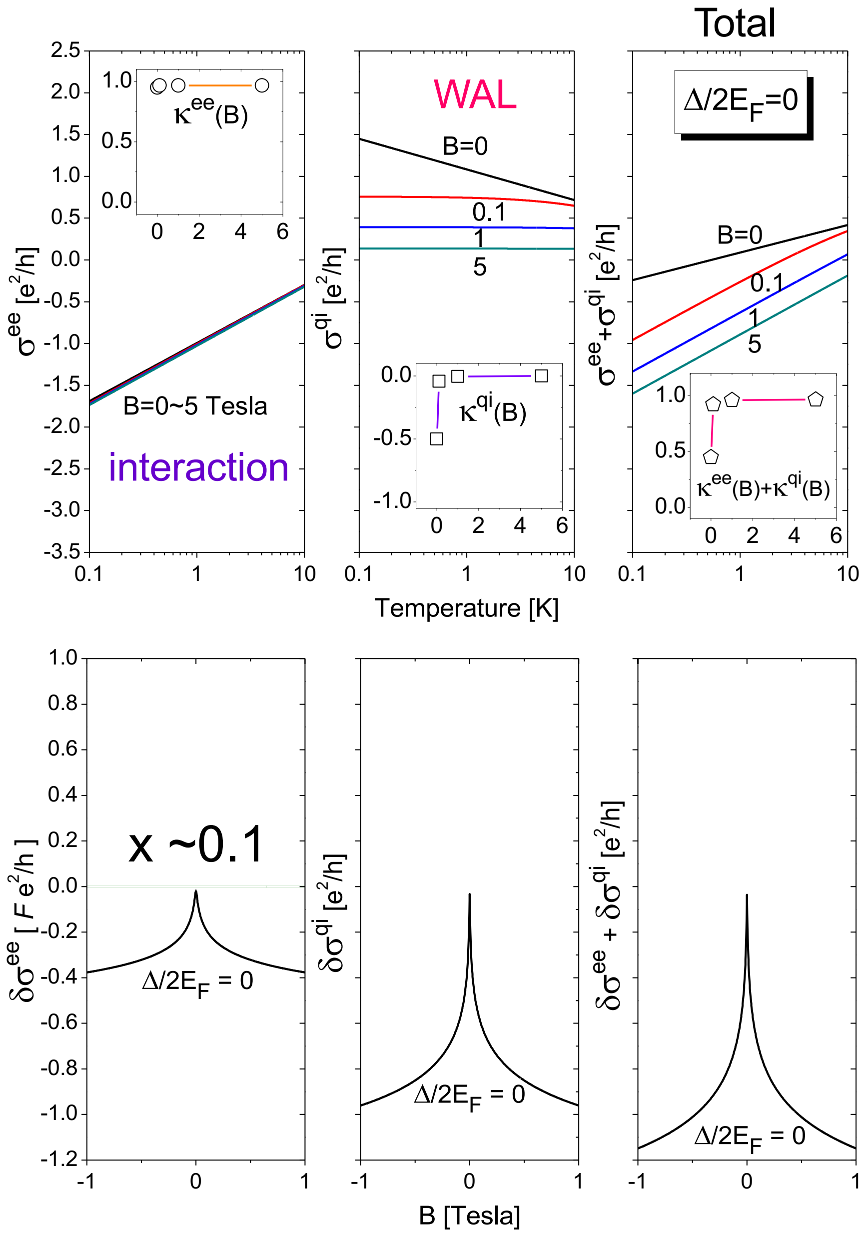
WAL

Zhang, Shuang Jia, Hasan, et al (PKU & Princeton), Nature Comm. 7, 10735 (2016)

Interaction effect

Interaction effect on Massless Dirac fermion



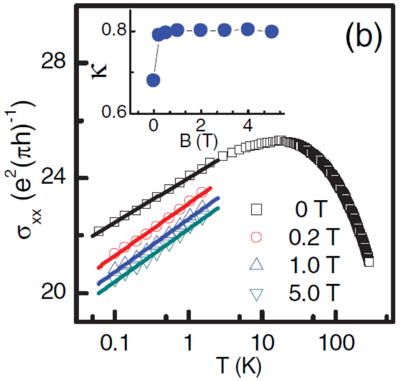


Interaction dominates temperature dependence

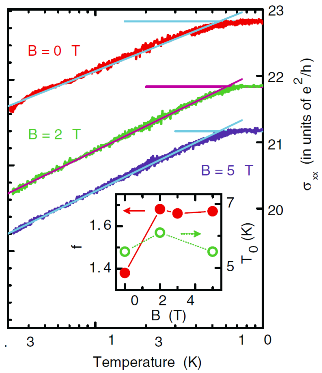
Quantum interference dominates magnetic field dependence

HZL & Shun-Qing Shen, PRL 112, 146601 (2014)

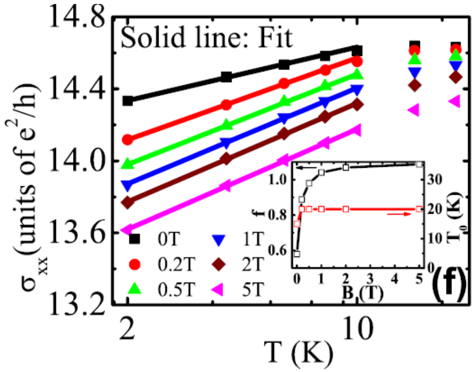
Slope of temperature dependence



Chen, Y.Q. Li, K. H. Wu, L. Lu, et al (IOP), PRB, 83, 241304 (R) (2011)
Bi2Se3



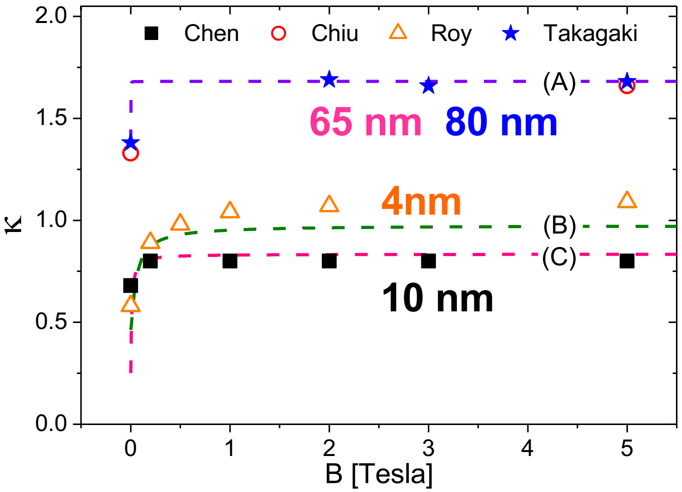
Takagaki et al (Berlin), PRB, 85, 115314 (2012)
Bi2Se3



Roy et al (Austin), APL 102, 163118 (2013);
Bi2Te3 ultrathin film

Slope

$$\kappa \equiv \frac{\pi h}{e^2}\frac{\partial \sigma}{\partial \ln T}$$



HZL & Shun-Qing Shen, PRL 112, 146601 (2014)

Summary

- ☐ Basics of Weak (Anti-) Localization
  - ✓ Symmetry classes
  - ✓ Berry phase
  - ✓ Interaction effects
  - ✓ Feynman diagrams
- ☐ Topological insulator thin films
  - ✓ WAL-WL Crossover in QAHE systems
  - ✓ Interaction-induced WL-like effect
- ☐ 2D Materials
  - ✓ Intervalley scattering
- ☐ 3D Topological semimetals
  - ✓ Weyl
  - ✓ Nodal-line

HZL & Shun-Qing Shen,
Chinese Physics B 25, 117202 (2016);
Frontiers of Physics 12, 127201 (2017)

Quantum transport in topological materials under magnetic fields

☐ Zero field

- ✓ Nonlinear Hall effect PRL 121, 266601 (2018); Nature 565, 337 (2019); NC 10, 3047 (2019)

☐ Weak field

- ✓ Weak anti-localization PRB 92,035203 (2015); PRB 93, 161110(R) (2016); NC 7, 10735 (2016); NC 7, 13142 (2016); PRL 122, 196603 (2019)
- ✓ Negative magnetoresistance NC 7, 10301 (2016); PRL 119, 166601 (2017)
- ✓ Majorana oscillation PRL 122, 147701 (2019)

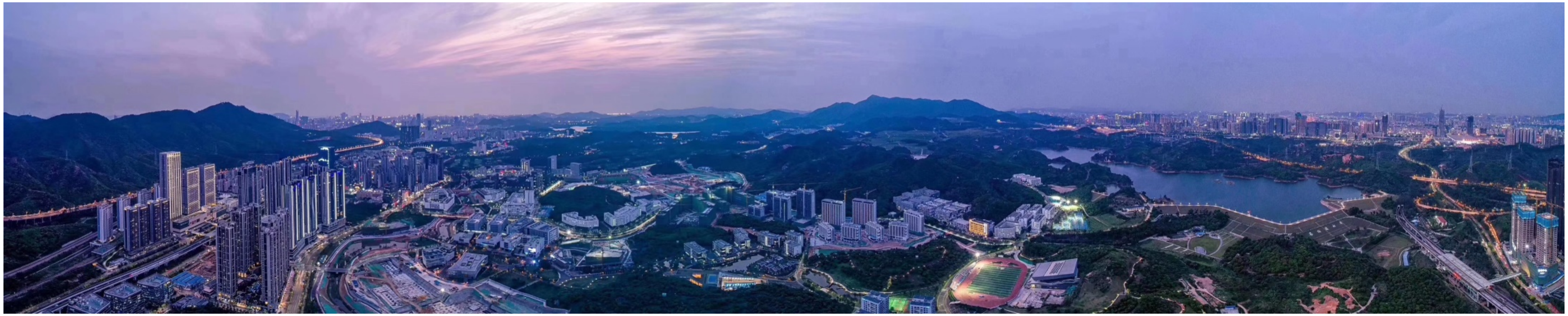
☐ Strong field

- ✓ 3D Quantum Hall effect PRL 119, 136806 (2017); Nature 565, 331 (2019)
- ✓ SdH oscillation PRL 117, 077201 (2016); PRL 120, 146602 (2018)

☐ Extremely strong field quantum limit

- ✓ Anomalous MR PRB 92, 045203 (2015); NJP 18, 053039 (2016);
- ✓ Weyl-node annihilation NP 13, 979 (2017)
- ✓ Distinguish topological insulators PRL 121, 036602 (2018)
- ✓ Non-saturating magnetization NC 10, 1028 (2019)

Thank you for your attention!



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- PhD

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luhaizhou@gmail.com